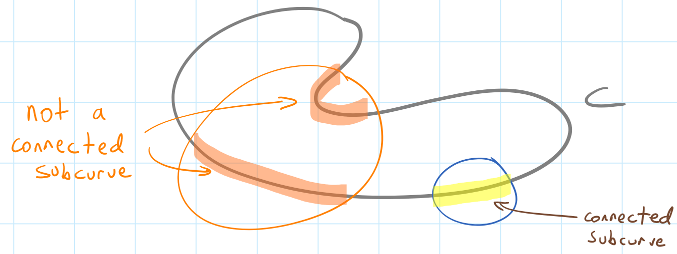
Exercise 5.43 "connected subcurve of C"

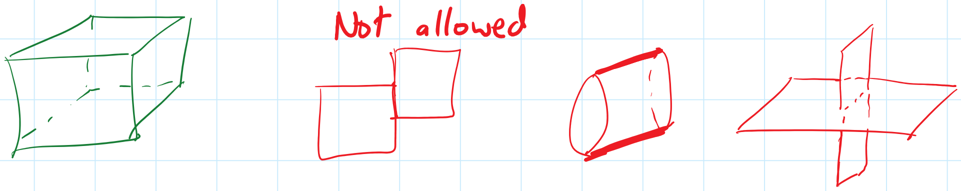


DEFINITION OF A POLYHEDRON

A polyhedron is composed of polygonal faces and satisfies:

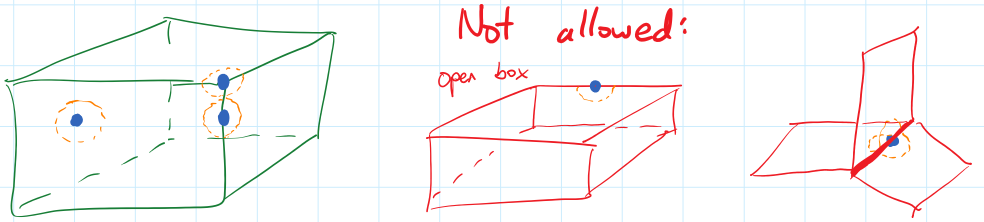
1. **Intersection Condition:** Any two faces may only intersect at a single vertex or along a common edge.

Not allowed



2. **Local Topology:** Around any point, a polyhedron looks like a face, an edge, or a corner.

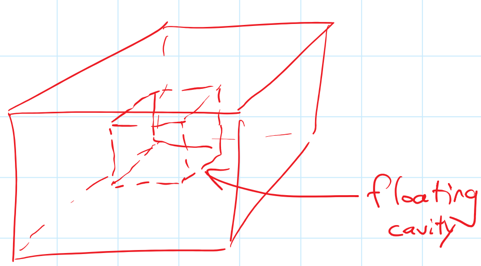
Not allowed:

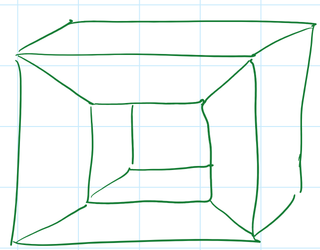


3. **Global topology:** Surface of the polyhedron must be connected.

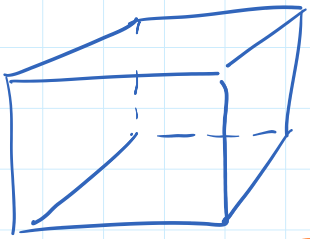
Not allowed:

Is allowed: tunnels OK

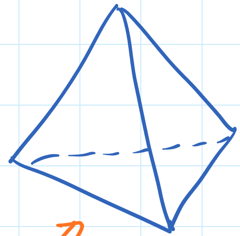




Exercise: Compute $V-E+F$ for some polyhedra

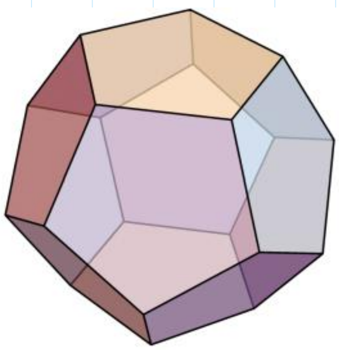


$V-E+F$
$=8-12+6$
$=2$

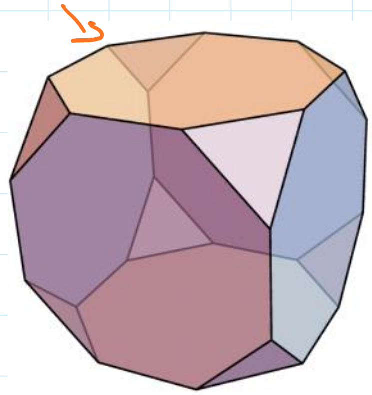


$V-E+F$
$=4-6+4$
$=2$

genus 0

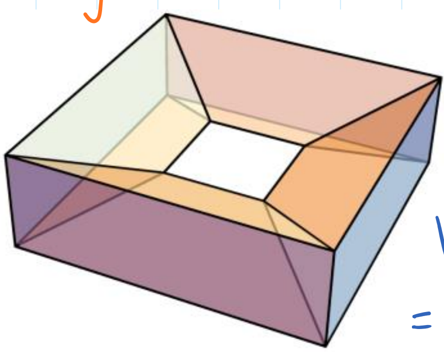


$V-E+F$
$=20-30+12$
$=2$



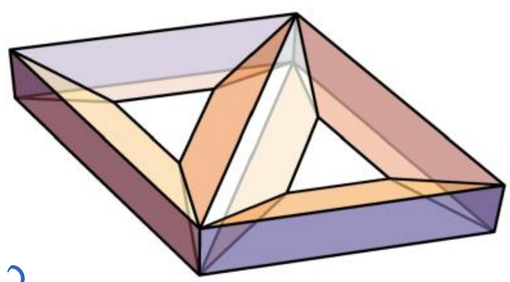
$V-E+F$
$=24-36+14$
$=2$

genus 1



$V-E+F$
$=12-24+12$
$=0$

genus 2



$V-E+F$
$=14-32+16=-2$

If polyhedron P has no holes/tunnels, then

$$V-E+F=2.$$

For each hole/tunnel, $V-E+F$ decreases by 2.

The quantity $V-E+F$ is called the EULER CHARACTERISTIC of a polyhedron P, denoted $\chi(P)$.

The number of holes/tunnels of a polyhedron is called the GENUS of the polyhedron.

THEOREM: If P is a polyhedron of genus g, then
$$\chi(P) = V-E+F = 2-2g.$$

1. Let P be a polyhedron of genus zero. If every face of P is either a pentagon or a hexagon, and if the degree of each vertex is 3, then how many faces are pentagons?

Suppose there are n pentagons and m hexagons.
Then:
$$F = n+m$$
$$2E = 5n+6m \quad \leftarrow \text{count the edges of each face}$$
$$3V = 5n+6m \quad \leftarrow \text{total degree}$$

Euler characteristic: since P has genus zero:
$$V-E+F = 2$$
$$6\left(\frac{5n+6m}{3} - \frac{5n+6m}{2} + (n+m)\right) = 2\cdot 6$$
$$10n + 12m - (15n+18m) + 6n + 6m = 12$$
$$n + 0m = 12$$
$$n = 12$$

12 pentagons