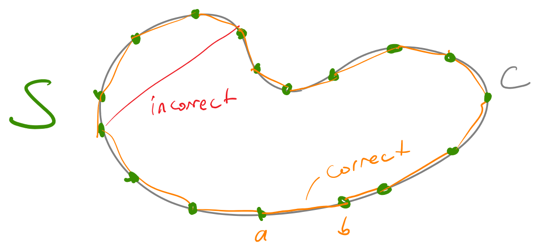
CURVE RECONSTRUCTION

Problem: Suppose we have a finite set of points S sampled from an unknown closed curve C.
How can we approximate C?



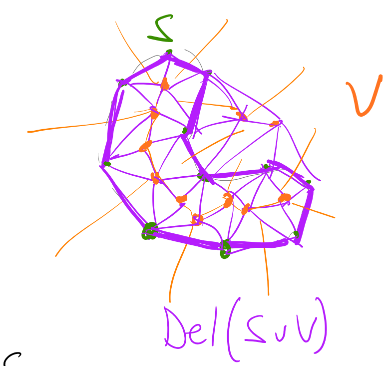
A **correct polygon reconstruction** P of curve C from sample S connect points a, b in P if and only if a and b are consecutive sample points along C.

CRUST Algorithm

relies on medial axis, Voronoi diagram, and Delaunay triangulation

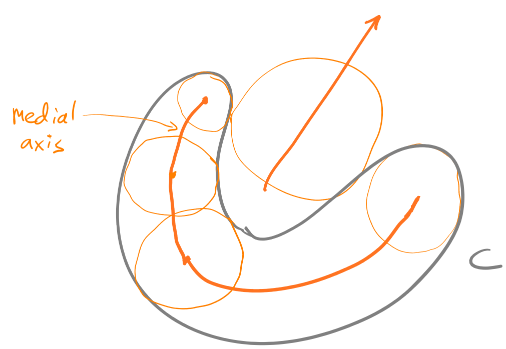
input: point set S, sampled from unknown curve C

1. Compute $Vor(S)$ and let V be the set of Voronoi vertices.
2. Compute Delaunay triangulation $Del(S \cup V)$
3. Polygon P is composed of the edges of $Del(S \cup V)$ with <u>both</u> endpoints in S.



Medial Axis of Curve C

Set of points that have two closest points on C



Key Insights:

(a) Voronoi vertices of Vor(S) lie near the medial axis of C.

Medial axis is equidistant from points on C.

Vor vertices are equidistant from 3 points on C

(b) Any circumscribing disk of incorrect edge of Del(S) crosses the medial axis of C.

Internal diagonals of Del(S) cross the medial axis or nearly so.

Correct edges of Del(S) are "far" from M(C)

Incorrect edges of Del(S) are "close" to M(C).

(c) An incorrect edge of Del(S) cannot also appear in Del(S ∪ V).

Empty circle property applied to Del(S):

a disk around an incorrect edge of Del(S) probably contains a point in V, so such edge cannot be in Del(S ∪ V).

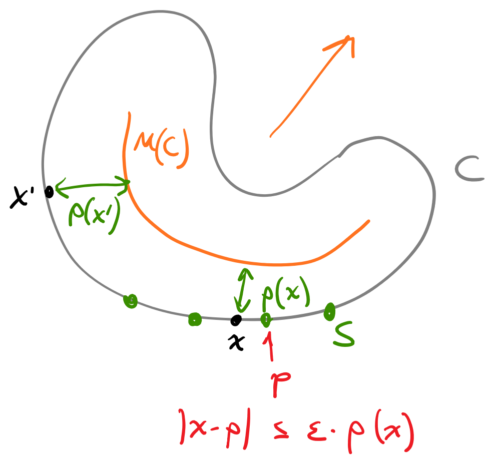
(d) A correct edge of Del(S) is in Del(S ∪ V)

If sample points are sufficiently "dense", then a disk around a correct edge of Del(S) will be away from the medial axis M(C).

Provable Correctness of CRUST Algorithm

Let x be a point on curve C. The local feature size $\rho(x)$ is the shortest distance from x to $M(C)$.

Let $0<\varepsilon<1$. A set of points S sampled from C is an ε-sample if each point $x \in C$ has a point $p \in S$ such that $|x-p| \le \varepsilon \cdot \rho(x)$.



THEOREM: The CRUST algorithm outputs the correct polygonal reconstruction of C whenever S is an ε-sample with $\varepsilon < \frac{1}{5}$.

NN-CRUST: provably correct for $\varepsilon < \frac{1}{3}$

nearest neighbor

Related algorithm: $\varepsilon < \frac{1}{2}$

Open problem: Find a provably correct alg. for $\varepsilon \ge \frac{1}{2}$.