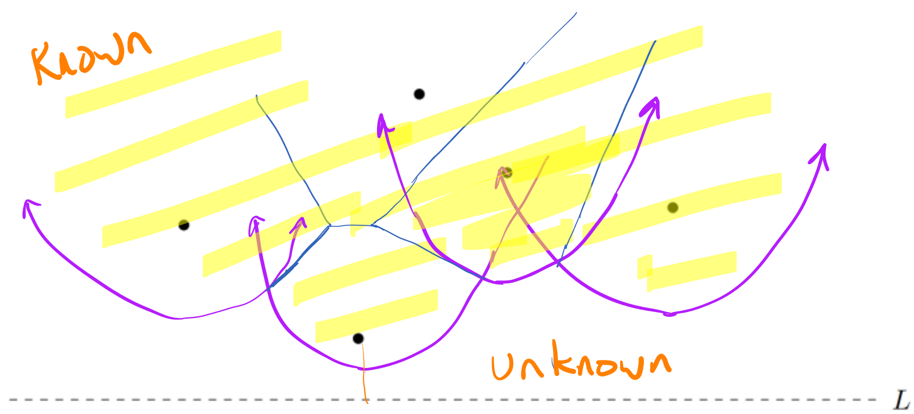
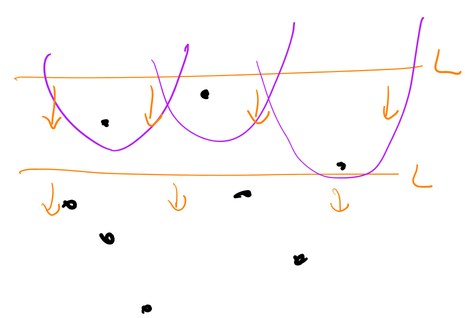
1



Which points are closer to existing sites than to line L?

? ? ?

FORTUNE'S ALGORITHM:



Line sweep algorithm

Sweep line L from top to bottom

maintain the boundary of the known diagram, and add new Voronoi vertices and edges.

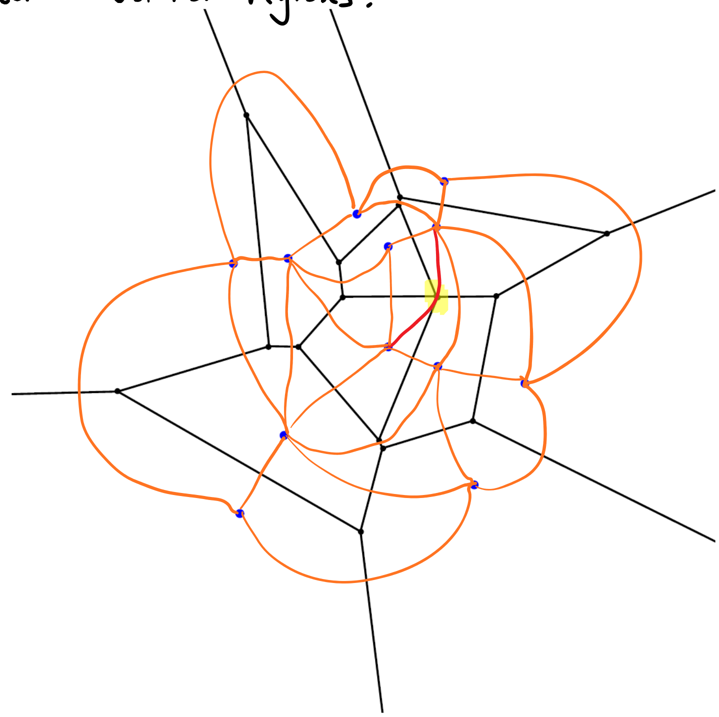
Maintain a priority queue of events, prioritized by y-coordinate:

- **Site events**: when sweep line crosses a site, then a new parabolic segment appears
- **circle events**: when sweep line is tangent to a circle through 3 sites, then record a Voronoi vertex

Complexity: $O(n \cdot \log n)$ n = number of sites

DUAL GRAPH: nodes are Voronoi sites

Two nodes are connected by an edge if their sites are in adjacent Voronoi regions.



OBSERVATIONS ABOUT DUAL GRAPHS OF VORONOI DIAGRAMS:

- If no four sites are cocircular, then dual graph is a triangulation.
- Is the dual graph the Delaunay triangulation? — empty circle property
  ... the minimum weight triangulation?
- Dual graph is planar: it can be drawn without edge crossings.

THEOREM: If S is a point set with no four sites cocircular, then the dual graph of $Vor(S)$ is the Delaunay triangulation of S.

proof: A point v is a Voronoi vertex if and only if there is a circle centered at v with three sites on it boundary and none in its interior.

Surrounding v are 3 Voronoi regions which are all adjacent, producing a triangle in the dual graph.

This triangle has its three vertices on the circle.

No other sites are in the circle, so the triangle satisfies the empty circle property of the Delaunay triangulation.

This holds for all triangles in the dual graph, so the dual graph is the Delaunay triangulation.

Convex Hulls, Voronoi Diagrams, and Delaunay Triangulations

The intersection of a paraboloid and most planes is an ellipse, which projects to a circle in the xy-plane.

PLAN: